A Clarification of the Derivation in Zhang and Boos (2023)

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This note concerns a recent preprint by Nicolas and Hotz (2025). While the preprint contains valuable results, I focus specifically here on their claim that there is an inconsistency in the derivation of the zero-surface-humidity temperature bound in Zhang and Boos (2023). I would like to clarify that the intent of our zero-humidity bound is not to represent the surface temperature when it is connected dry-adiabatically to 500 hPa (as considered by Nicolas and Hotz, 2025), but rather to the top of the atmosphere (or the tropopause, neglecting humidity there). A more detailed explanation of this intent is provided on the next page.

Additionally, Zhang and Boos (2023) also introduced an adjusted bound, obtained by subtracting the zero-humdity bound with a climatological minimum summer surface humidity $q_{\min}$ (scaled by L_v/c_p). This adjusted bound is more practically relevant in interpretaing regional observations (see Figs. 3d–f and Section "Insight into Recent Heatwaves") and demonstrates the utility of a moist-convective scaling for dry heatwaves. I agree with Nicolas and Hotz (2025) that $q_{\min}$ must satisfy $q_{\min} > q^*_{500}$; however, the adjusted bound, despite being similar to the moist bound in Nicolas and Hotz (2025), may not have been discussed in the preprint.

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Detailed explanation

Zhang and Boos (2023) presents a top-down control on surface temperature from an approximately moist-adiabatic free troposphere in midlatitude summer, a mechanism that has received relatively little attention in the context of dry midlatitude heatwaves. There are multiple perspectives from which a surface temperature bound can be derived, including the textbook dry convective neutrality perspective and the surface energy budget perspective. In contrast, the temperature bounds in Zhang and Boos (2023) are intended specifically to represent the top-down control of the free troposphere. To this end, a key assumption stated in the Physical Mechanism and Theory section is that MSE^*_{500} serves as a proxy for the approximately uniform MSE^* above the LCL, which gives

$$\mathrm{MSE}^*_{500}(T_{500}) \approx \mathrm{MSE}^*_{\mathrm{tp}}, \tag{R1}$$

where "tp" denotes the tropopause. Neglecting saturation vapor pressure at the tropopause, this implies

$$c_p T_{500} + L_v q^*_{500} + g z_{500} \approx c_p T_{tp} + g z_{tp}. \tag{R2}$$

With the bulk stability constraint $\mathrm{MSE}_s \leq \mathrm{MSE}^*_{500}$, we obtain

$$c_p T_s + g z_s \leq c_p T_{500} + L_v q^*_{500}(T_{500}) + g z_{500}(T_{500}) - L_v q_s. \tag{R3}$$

Obtaining a uniform bound. Zhang and Boos (2023) has made justifiable approximations to z_{500} in order to write the first three terms as functions of a single variable (T_{500}). Still, the variety of q_s across regions and events prevents a uniform bound. The fewer unknowns, the more useful a theory. We then amplify the right-hand side—that is, by minimizing q_s—thereby preserving the inequality and obtaining a uniform bound independent of q_s. Apparently $q_s \geq 0$, therefore setting $q_s = 0$ leads to

$$c_p T_s + g z_s \leq c_p T_{500} + L_v q^*_{500}(T_{500}) + g z_{500}(T_{500}). \tag{R4}$$

This yields the zero-humidity bound in Zhang and Boos (2023).

The zero-humidity bound is perhaps best viewed as the limiting case of the adjusted bound also proposed in Zhang and Boos (2023). Also following the approach of minimizing q_s to a uniform value, we minimize q_s in Eq. (R3) to a climatological summertime minimum humidity q_{min} which gives

$$c_p T_s + g z_s \leq c_p T_{500} + L_v q^*_{500}(T_{500}) + g z_{500}(T_{500}) - L_v q_{\mathrm{min}}. \tag{R5}$$

Because q_{min} varies across regions, this adjusted bound can only be demonstrated regionally. This is shown for Western Europe, Russia, and the Pacific Northwest (Figs. 3e–f).

Physical picture. Zhang and Boos (2023) describes the following progression: the free troposphere warms, creating an inversion; the boundary layer then warms to reduce this inversion, bringing the temperature profile closer to an adiabat, with a moist adiabat aloft connected to a dry adiabat below; if the anticyclone persists, continued surface drying deepens the dry-adiabatic portion of the profile, which progressively erodes into the moist-adiabatic portion and allows further surface warming until the anticyclone moves away. The progression of vertical temperature profiles for three record heatwaves in three aforementioned regions (Fig. S6) roughly aligns with this conceptual picture. The zero-humidity bound would then correspond to the limiting case in which this process continues until the moist layer vanishes and the column becomes entirely dry-adiabatic. This limit may not be realistic, but it is physically consistent.

Connection to textbook dry adiabats. Comparing to a dry adiabat connecting the surface to 500 hPa, which gives

$$c_p T_s + g z_s \leq c_p T_{500} + g z_{500}, \tag{R6}$$

the zero-humidity bound in Zhang and Boos (2023) yields

$$c_p T_s + g z_s \leq c_p T_{\mathrm{tp}} + g z_{\mathrm{tp}}, \tag{R7}$$

corresponding to a dry adiabat connecting the surface to the tropopause. This distinction follows from Eq. (R1) which reflects the assumption that MSE^*_{500} serves as a proxy for the approximately uniform MSE^* above the LCL.

Closing remarks. To summarize, 1) the inclusion of q^*_{500} in the zero-humidity bound corresponds to a different dry adiabat–one that connects the surface to the top of the atmosphere (or the tropopause), rather than to the 500 hPa level. 2) The zero-humidity bound is a singular limiting case of the adjusted bound with surface moisture that was also proposed in Zhang and Boos (2023). 3) This inclusion of q^*_{500} in both bounds is a deliberate choice to balance parsimony and realism.

I acknowledge that aspects of the framing and presentation could lead to confusion, and I am happy to clarify. I am also happy to acknowledge challenges to our methodology comparing theory with observations and deviations that arise (a discussion on this may be provided in a future note or paper). That said, the inclusion of q^*_{500} in our derivation is not an oversight.